Figure 3 illustrates a two-dimensional representation of the multi-commodity flow of F_1 and F_2 of Figure 2. The MFF of this two-dimensional space is bounded by the plane 240 and the surface of cube 230. To construct the MFF, a sample point 300 is determined using, for example, the previously discussed Garg and Konemann algorithm. An approximate MFF (AMFF) is determined by linearly joining points f2 210 and sample point 300 with a straight line 330 and joining sample point 300 to point f1 200 with a straight line 320. A large number of AMFFs can be drawn through points f1, f2 and sample point 300. Thus, a piece-wise linear continuous, AMFF is constructed that may be used to quickly determine flow rates when the price vectors 260 or 280 change.

In another embodiment, a polynomial of order greater than one can be drawn through the three illustrated points. Curve 350 (of Figure 3) represents a polynomial of order greater than one connecting end points f1, f2 and sample point 300. In another embodiment, an AMFF may be constructed using a plurality of polynomials of order greater than one to connect the individual end and sample points. As illustrated in Figure 3, an AMFF is constructed using curve 310 and curve 340, which are generated by polynomials of order greater than one and the polynomial surfaces are generated by spline functions wherein the second derivation of said spline functions. To provide a seamless transition between such polynomials, the polynomials are selected such that the value of their second derivative at a contacting sample point are equal.

Figure 4 illustrates an AMFF constructed using two sample points 400 and 410 and the two end points f1 200 and f2 210. The AMFF in this embodiment is constructed using polynomials of order one, line segments 440, 430 and 420 between points f2 210

and sample point 410, between sample points 410 and 400, and between sample point 400 and point f1 200, respectively.

An error bound can be determined by extending line segments 440, 430 and 420. For example, the error between the piece-wise linear AMFF and the MFF is contained within the triangular area having the three vertices, sample points 400, 410 and point 470, which is formed at the intersection of the extended line segments 420, 440. Similarly, the MFF is bound by the triangular area having end point f2 210 and sample point 410 as two vertices and a third vertex at point 480, which is the intersection of the extended line segment 430 and the maximum single commodity flow along flow F_2.

In Figure 5, five sample points 510, 520, 530, 540 and 560 are positioned along minimal AMFF at spacing defined by Δv. This spacing is chosen to reduce the maximum error between a MFF and an AMFF. From each of sample points 510, 520, 530, 540 and 560, a direction of an identifying characteristic may be obtained. Direction is determined by specifying an angular displacement from a reference axis. A direction of an identifying characteristic between point 510 and the origin may be specified by the angular displacement, α_1. Similarly, displacements α_2 and α_3 specify the direction of an identifying characteristic from points 520, 530 respectively.

Sample points along the MFF may be determined based on the directions of the identifying characteristic using, for example, the Garg and Konemann algorithm, as previously discussed. Sample points 570, 580, 590, 600 and 610, which lie on the MFF, correspond to point 510, 520, 530, 540 and 560, respectively. An AMFF may then be constructed between end points f_1, 200 and f_2, 210 and sample points 570, 580, 590, 600

and 610. In this example, the AMFF is constructed using linear segments. The AMFF can be used to determine data flow factors at points other than the sample points 570, 580, 590, 600 and 610 with known values of errors introduced.

A flow chart of the method of the invention is shown in Figure 6. Starting at Block 600, the network information, such as nodes location, length and available capacities of the links, is acquired at Block 620. Some sample points f_1, f_2,...f_m are computed at Block 630 to determine the maximum revenue flows for some interested and fixed prices, p_1, p_2,..., p_m. These sample points can be computed off-line using known algorithms, such as the Garg and Konemann algorithm. In Block 640 the AMFF is constructed. An AMFF is an continuous and convex curve passing through these given sample points, $\mathbf{f}_1$, $\mathbf{f}_2$..., f_m. A simple AMFF is the piece-wise linear plane that passes through these sample points. The price data are obtained repeatedly at block 650. The price vector **p** (t) may change with time t dynamically – *e.g.*, it may change a lot during a short period, such as a single day, while the network may remain unchanged during months, or even years.

To track the maximum revenue while p(t) varies with time, the AMFF can be reused. The AMFF may be constructed, as previously discussed, using a piece-wise linear approximation or using polynomials of order greater than one. In Block 660 the method of the invention operates to adjust and reallocate the flows while the price vector changes, such that the maximum revenue are realized. This can be done by adjusting the flow to the point on the AMFF which is perpendicular to the price vector **p**(t). Alternatively the flow can simply be adjusted to the $\mathbf{f}_i$ in which $\mathbf{p}_1$, $\mathbf{p}_2$..., $\mathbf{p}_m$, is the closest to **p**(t) in the case where the AMFF is difficult to construct.